

The quantization of exotic states in $SU(3)$ soliton models: A solvable quantum mechanical analog

Aleksey Cherman, Thomas D. Cohen, Abhinav Nellore
Department of Physics, University of Maryland, College Park, MD 20742-4111

The distinction between the rigid rotor and Callan-Klebanov approaches to the quantization of $SU(3)$ solitons is considered in the context of exotic baryons. A numerically tractable quantum mechanical analog system is introduced to test the reliability of the two quantization schemes. We find that in the equivalent of the large N_c limit of QCD, the Callan-Klebanov approach agrees with a numerical solution of the quantum mechanical analog. Rigid rotor quantization generally does not. The implications for exotic baryons are briefly discussed.

I. INTRODUCTION

Recent experimental reports[1] of the observation of an exotic θ^+ baryon have rekindled interest in the quantization of exotic states in $SU(3)$ soliton models[2]. The soliton models are unique among the theoretical tools used to analyze these reported states because they predate the experiment and predict the mass of the state very closely to the claimed experimental values [3, 4].

A commonly employed technique for quantizing solitons treats the system as a rigid rotor in which the soliton collectively rotates, with its internal degrees of freedom fixed from the classical solution[5, 6]. This technique is clearly correct for $SU(2)$ chiral solitons when treated in the large N_c limit [7] (where N_c is the number of colors in QCD). However, the validity of applying the rigid rotor approach to the quantization of $SU(3)$ solitons has been questioned in situations where the Wess-Zumino term can play an active role in the dynamics of the system — as it does for exotic baryon states [2, 8, 9, 10]. This issue remains controversial [2, 8, 9, 10, 11]. In this paper we study a numerically tractable quantum mechanical system that has many critical features in common with $SU(3)$ solitons to obtain insight about the underlying issues.

It should be noted that there are *two* major approaches in the literature for quantizing $SU(3)$ solitons. One is the rigid rotor approach mentioned above, in which the collective rotational and vibrational modes of the soliton are assumed to be decoupled, and only the rotational modes are quantized[6]. This approach has a number of benefits: it is relatively simple to implement and is a straightforward generalization of the Adkins, Nappi and Witten procedure which is known to be justified at large N_c for the case of $SU(2)$ solitons[7]. Moreover, it is known to be justified at large N_c for non-exotic collective states in $SU(3)$ models. The vast majority of the

published studies of exotic baryons in the context of soliton models have used this approach. The other approach to quantizing $SU(3)$ solitons is the Callan-Klebanov approach [9, 12]. This scheme is most easily understood in the case of broken $SU(3)$, in which case excitations carrying strangeness are unambiguously vibrational states, and should be quantized as harmonic vibrations. It has been argued that this method remains valid as one approaches the $SU(3)$ limit [9]. In fact, for the non-exotic states one of the vibrational modes becomes softer and goes to zero frequency as the $SU(3)$ limit is approached, and thus reproduces the results of rigid rotor quantization. Therefore it seems that this method should be valid for both broken and unbroken $SU(3)$. However, for exotic states the Callan-Klebanov approach does *not* reproduce the rigid rotor result; indeed when applied to the original Skyrme model it gives *no* exotic resonant states at all[9]. The Callan-Klebanov approach has the obvious disadvantage of being more difficult to implement; this may explain the fact that it has not been widely used in studies of exotic baryon states. Indeed, to the best of our knowledge only two such calculations have been reported[9, 13].

The fact that the Callan-Klebanov method gives different results from rigid-rotor quantization for exotic states implies that at least one of these methods is wrong. Assuming that one is correct, it is critical to know which one. It has been argued elsewhere on a number of grounds that the rigid-rotor approach is the culprit[2, 8, 9, 10], and the Callan-Klebanov method is correct, at least at large N_c . While there have been attempts in the literature to rebut at least some of these arguments [11], it has been argued that these rebuttals are fundamentally flawed[8]. We will not attempt to recap these arguments but instead refer the reader to the original literature.

Our purpose here is simply to consider a tractable quantum mechanical system that has states analogous to the ex-

otic states of the $SU(3)$ soliton, which is analytically intractable. This system can be solved numerically (to essentially any desired degree of accuracy) and via both approximate methods — the Callan-Klebanov approach and the rigid-rotor approach in the analog of the large N_c limit. One can then explicitly see which approach works. As we will show below, the Callan-Klebanov method reproduces the numerical solutions for this model up to expected errors of order $1/N_c$, while the rigid-rotor approach generally fails.

The model considered here was introduced in refs. [8, 10]. In those works it was observed that the Callan-Klebanov approach and rigid-rotor approach gave different answers for the excitation spectrum of the model, and it was argued on semiclassical grounds that the assumptions underlying the rigid-rotor quantization were not self consistent. However, these works did not show explicitly that the Callan-Klebanov approach actually produces the correct spectrum, and that the rigid rotor approach does not.

This paper is organized as follows. In the next section, the model will be introduced. The two subsequent sections will implement the rigid rotor quantization and the Callan-Klebanov quantization for this model. (Some details of the Callan-Klebanov treatment are relegated to an appendix). The next section contains a brief discussion of the numerical solution of the model and a comparison of the numerical solution with the two approximation methods. Finally, we discuss the implications of our results for soliton treatments of exotic baryons.

II. A TRACTABLE MODEL

We wish to study a numerically soluble model that incorporates the relevant features of the excited states of $SU(3)$ solitons. Given the nature of the critique of the rigid rotor treatment [2, 8, 9, 10], in order to mimic the soliton problem, we need a system in which there are both collective rotational and vibrational degrees with the same quantum numbers. There should also be a force on the system that is topological in nature and velocity dependent to mimic the effect of the Wess-Zumino term (which is topological and first order in time and thus acts on velocities).

To do this, let us consider the problem of a composite charged particle moving nonrelativistically on the surface of a sphere of radius R , which has a magnetic monopole of strength g at its center. It was observed long ago by Witten that the motion of a charged particle on the surface of a sphere in the field of a magnetic monopole is topological

in essentially the same way as the motion of chiral fields in the presence of the Wess-Zumino term[14]. Indeed, the original rigid rotor quantization of $SU(3)$ solitons by Guadagnini was explicitly done in analogy to the monopole problem[5]. It is important that we consider a composite system, *i.e.*, one with internal degrees of freedom. The dynamics of the composite system are analogous to the internal dynamics of the soliton and the key issues are associated with the possible interplay of internal and collective degrees of freedom.

To be concrete, we consider our composite particle as being made of two point-like constituent particles[8, 10]. One constituent is a charged particle (with charge q). The other constituent particle is electrically neutral. We take both constituent particles to have the same mass (M). The particles interact via a nonsingular potential that binds the particles together. To ensure that the system is rotationally invariant, the potential can depend only on the separation between the particles. Since the particles are strongly bound, and thus spend most of their time near the minimum of the potential, we can take the interaction to be due to an approximately harmonic potential of spring constant k . As noted above, the magnetic field due to the monopole serves to provide the desired velocity-dependent topological force. The analog of the classical static soliton is simply the classical configuration which minimizes the energy—namely, the two particles on top of each other at the minimum of the potential. This configuration will be referred to as the “soliton”.

The semiclassical treatment of the $SU(3)$ soliton is only justified in the large N_c limit of QCD. Thus, it is important that the various parameters in the toy model are chosen to scale with N_c in a manner that emulates the soliton case:

$$q \sim N_c^0 \quad R \sim N_c^0 \quad g \sim N_c^1 \quad M \sim N_c^1 \quad k \sim N_c^1 \quad . \quad (2.1)$$

These scaling rules ensure that energy of the classical “soliton” scales as N_c^1 , the characteristic frequencies associated with internal excitations of the “soliton” ($\sqrt{2k/M}$) scale as N_c^0 , and that the excitations associated with exotic motion also scale as N_c^0 . This behavior is analogous to the $SU(3)$ soliton system [8].

III. RIGID-ROTOR QUANTIZATION

To develop some intuition about the rigid-rotor approach, first consider a simpler problem: the charged composite particle moving on the surface of a sphere *without* a magnetic monopole. Both constituents of the particle have mass M and interact via a (nearly) harmonic potential with spring

constant k . The classical ground state of the “soliton” has the two particles on top of one another located at some point (say the north pole). Of course, since the “soliton” breaks rotational symmetry, this classical state is highly degenerate—the “soliton” can be localized at any point on the sphere. Thus, there exists a collective manifold of configurations, all with the same classical ground state energy.

One can consider the system slowly moving through this manifold—*i.e.*, the two particles move together in lock-step, with the center of mass moving collectively around the sphere. Without solving the quantum equations of motion one can see that this approximation is justified quantum mechanically in the large N_c limit due to the large spring constant and the large mass in Eq. (2.1). To see this, first one assumes that the intrinsic vibrations are decoupled from the collective motion, and subsequently checks for self consistency. The characteristic energy scale for low-lying collective excitations is just the inverse of the moment of inertia $E_{\text{rot}} \sim \frac{1}{2MR^2} \sim 1/N_c$. This is very small compared to the characteristic energy associated with the intrinsic vibrations $E_{\text{vib}} \sim \sqrt{2k/M} \sim N_c^0$. The fundamentally different scales at large N_c allow the collective rotational motion to be essentially decoupled from the intrinsic vibrational motion. Thus the composite system—our “soliton”—behaves as if it were a single charged particle of mass $2M$ at large N_c . Indeed this is hardly surprising: the strong spring constant ensures that in the large N_c limit the two particles are tightly bound, and thus move together collectively. This is rigid rotor quantization since the internal structure of the “soliton” is approximated as being rigid and corrections to this are higher order in $1/N_c$.

Now consider what happens when the monopole field is added to the system. The spring constant remains strong and the wave function for the composite system remains highly localized. Thus it is plausible that the two particles continue to move collectively together in the same way that they did in the absence of the monopole. This reasoning suggests that the collective excitations will be those of a single particle of mass of $2M$ and charge q moving in the field of the monopole.

To obtain the collective energy spectrum, we need only consider the dynamics of a single charged particle (of mass $2M$ and charge q) on a sphere of radius R with a magnetic monopole of strength g at its center. This system is well described in refs. [5, 14]. Any point on a sphere can be labeled by an element of $SU(2)$. (Technically, a point on a sphere S^2 corresponds to a fiber in $SU(2)$; elements in a fiber are related by unitary phases.) Thus the states of a single particle on a sphere correspond to the irreducible representations

of $SU(2)$, and these can be written in terms of Wigner D-functions $D_{m,m'}^J$. Without a magnetic monopole, a particle sitting on a sphere has no intrinsic angular momentum and the m' of the Wigner D-function is always zero. The allowed states are then simply the spherical harmonics Y_m^J . However, the presence of a magnetic monopole gives the particle an intrinsic angular momentum of qg . The reason for this is simple. A system which has both an electric and magnetic field has momentum carried in the fields. A calculation of $\vec{J}_{\text{field}} = \vec{r} \times \vec{p}_{\text{field}}$ for a static classical field configuration yields $\vec{J} = qg\hat{r}$ —the angular momentum along the intrinsic z axis is qg . This restricts the allowed representations to those where $J \geq qg$. Thus the states of the particle can be written as Wigner D-functions $D_{m,qg}^J$, with $J \geq qg$.

It is easy to find the energy of the system. At the classical level, the Hamiltonian is given by

$$H = \frac{J^2 - (qg)^2}{2I}. \quad (3.1)$$

where I is the moment of inertia. The energy is purely kinetic and is zero when the system is at rest, namely, for $J = qg$. For the present system $I = 2MR^2$ where the factor of two reflects the fact that the mass of the composite is $2M$. To quantize the system one simply promotes J to a quantum operator. The energy of the system is then given by

$$E(J) = \frac{J(J+1) - (qg)^2}{4MR^2} \quad \text{with } J \geq qg. \quad (3.2)$$

The excitation energy of the lowest-lying collective state is then given by

$$\Delta E_R \equiv E(qg+1) - E(qg) = \frac{qg}{2MR^2}. \quad (3.3)$$

IV. CALLAN-KLEBANOV QUANTIZATION

There is another way to look at the problem. First, consider the case of a single charged particle moving on a sphere in the field of a strong magnetic monopole from a classical perspective. The system is a charged particle moving in a strong magnetic field. The paths of particles moving in magnetic fields bend, and in strong fields they bend into classical orbits of small radius. Thus, the particle does not make great circle orbits around the sphere but instead makes tight local orbits. Moreover, it is well known that the quantization of these localized orbits leads to Landau levels—namely, the excitation spectrum of a harmonic oscillator [15]. Now consider

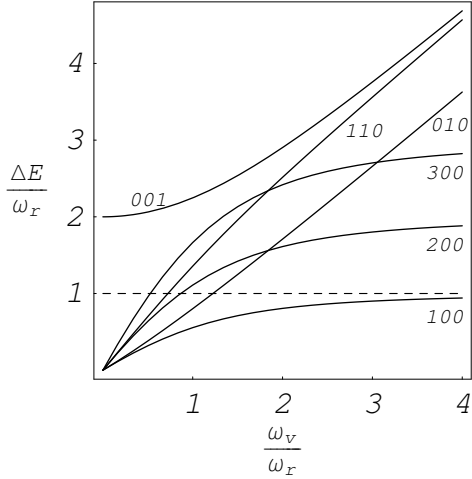


FIG. 1: Semiclassical solutions of the system based on the Callan-Klebanov approach as given in [8]. The energy is measured in units of ω_r and the solutions are plotted as a function of the ratio ω_v/ω_r , with $\omega_r = \frac{qg}{2MR^2}$ and $\omega_v = \sqrt{\frac{2k}{M}}$. The plot shows $\Delta E = E - E_{ground}$. Each solution is labeled by the $n_1 n_2 n_3$ of Eq. (4.3), giving its decomposition in terms of the three non-zero positive normal modes of the system.

the classical dynamics of the complete toy model, with two interacting particles on a sphere in the presence of a magnetic monopole. There are two types of harmonic dynamics: the orbits associated with motion of the center of mass in the magnetic field, and the motion associated with excitations of the two particles relative to one another. These two types of motion each have characteristic frequencies associated with them. The orbits of the center of mass due to the magnetic field have a characteristic frequency,

$$\omega_r \equiv \frac{qg}{2MR^2}, \quad (4.1)$$

which is the cyclotron frequency of a particle of mass $2M$ and charge q moving in the magnetic field of a monopole of strength g a distance R away. It is worth observing that ω_r is precisely equal to the excitation energy in the rigid rotor approximation of Eq. (3.3). The characteristic frequency of

the intrinsic vibrations is

$$\omega_v \equiv \sqrt{\frac{2k}{M}}. \quad (4.2)$$

The key point is that the classical equations of motion can induce mixing between these two types of motion.

The classical equations can be truncated at harmonic order and then solved for the normal mode frequencies. Due to the presence of velocity-dependent forces, it is useful to formulate the problem in terms of coupled first-order differential equations for positions and velocities. This classical problem was analyzed in refs. [8, 10] and this analysis is briefly recapitulated in the appendix. The problem has four degrees of freedom and hence there are four normal mode frequencies (which come paired as positive and negative frequency solutions to the equations of motion). Three of the normal mode frequencies are nonzero and the fourth is a zero mode. The Callan-Klebanov approach amounts to the quantization of these harmonic modes.

The zero mode is non-dynamical in nature: it corresponds to relocating the position of the “soliton” to a new point on the sphere but has no velocity associated with it. Quantum mechanically this mode corresponds to the nonexotic states of the system. For the present case it represents the “excitation” of one of the $2qg + 1$ degenerate ground states. In the context of the $SU(3)$ soliton case it corresponds to the excitation of the usual hyperons from the nucleon (which, of course, are exactly degenerate in the $SU(3)$ limit).

The non-zero modes correspond to physical harmonic excitations. The excitation spectrum to leading order in the $1/N_c$ expansion is thus given by

$$\Delta E = E - E_{ground} = n_1 \omega_1 + n_2 \omega_2 + n_3 \omega_3 \quad (4.3)$$

where the ω_i are the three non-zero positive eigenfrequencies. In Fig. 1, we plot a few representative low-lying states. In this figure ΔE is given in units of ω_r and it is plotted as a function of differing spring constants which are reflected in the dimensionless ratio $\frac{\omega_v}{\omega_r} = \frac{R^2 \sqrt{8Mk}}{qg}$. The advantages of working with these two dimensionless ratios should be clear; the results only depend on particular combinations of the parameters greatly simplifying the analysis. Thus, it is sufficient to work with fixed values of M , R , q and g while varying k to explore all of the physics at large N_c . It is worth noting that $\frac{\omega_v}{\omega_r}$ is of order N_c^0 and thus the large N_c limit can be taken for any value of this ratio.

For the sake of comparison, in Fig. 1 we also plot the excitation energy of the collective state predicted by the rigid

rotor quantization as a dashed line (located at $\frac{\Delta E}{\omega_r} = 1$). One key observation needs to be made at this point. Clearly these two approaches predict different excitation spectra for generic values of $\frac{\omega_v}{\omega_r}$. The two approaches do agree in the limit $\frac{\omega_v}{\omega_r} \rightarrow \infty$ where $\omega_1 \rightarrow 1$, but as noted above nothing in the large N_c scaling rules tells us that this ratio should be large.

It is apparent from this analysis that there are two conflicting pictures for quantization in this problem. At this stage it is worth noting that the rigid-rotor approach describes far fewer states in the spectrum than the Callan-Klebanov approach does. It is important to determine which of these two quantization schemes actually describes this system as the large N_c limit is approached.

V. NUMERICAL SOLUTION OF THE MODEL

To numerically solve the quantum system one must specify the Hamiltonian in a particular basis. As shown in ref. [5], Wigner D-functions with $J \geq eg$ form a basis of states for the charged particle. The states in this basis are labeled by J ($J \geq qg$), m_J ($J \geq m_J \geq -J$) and $m'_J = qg$. Of course, the standard spherical harmonics form a basis of states for the neutral particle and are labeled by L and m_L ($L \geq m_L \geq -L$). We can obtain a basis of states for the composite system by taking tensor products of the basis states for the charged and neutral particles.

In the semiclassical analysis it is sufficient to consider a harmonic potential in the large N_c limit; anharmonic effects only come in as $1/N_c$ corrections. Thus in the numerical

model studied, the interaction potential between the particles should be approximately quadratic at short distances to mimic the soliton problem. However, since the problem is posed on a sphere and we are using angular variables, it is necessary that the potential be periodic in the angular separation. The simplest form for the potential with these properties is $kR^2(1 - \cos \gamma)$, where γ is the angular separation between the charged and neutral particles.

The Hamiltonian for this system is the sum of the kinetic energies of the charged particle, the kinetic energy of the neutral particles and the interaction potential between them:

$$\hat{H} = \hat{H}_q + \hat{H}_n + \hat{H}_{\text{int}}. \quad (5.1)$$

The charged particle kinetic energy is

$$\hat{H}_q = \frac{\hat{J}^2 \otimes \hat{1}_n - (eg)^2}{2MR^2}, \quad (5.2)$$

for the reasons discussed earlier. The neutral particle term is

$$\hat{H}_n = \frac{\hat{1}_q \otimes \hat{L}^2}{2MR^2} \quad (5.3)$$

and the interaction term is

$$\hat{H}_{\text{int}} = R^2 k (1 - \cos \gamma). \quad (5.4)$$

A straightforward computation using standard identities produces the matrix elements of the total system Hamiltonian:

$$\begin{aligned} \langle J', m'_J, eg; L', m'_L | \hat{H} | J, m_J, eg; L, m_L \rangle = & \frac{J(J+1) + L(L+1) - (eg)^2}{2MR^2} \delta(J, J') \delta(m_J, -m'_J) \delta(L, L') \delta(m_L, -m'_L) + \\ & kR^2 \delta(J, J') \delta(m_J, -m'_J) \delta(L, L') \delta(m_L, -m'_L) - (-1)^{eg - m'_J + m'_L} \sqrt{(2J+1)(2J'+1)(2L+1)(2L'+1)} \times \\ & \sum_{c=-1}^1 \begin{pmatrix} J' & 1 & J \\ -eg & 0 & eg \end{pmatrix} \begin{pmatrix} J' & 1 & J \\ -m'_J & -c & m_J \end{pmatrix} \begin{pmatrix} L' & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L' & 1 & L \\ -m'_L & c & m_L \end{pmatrix} \end{aligned} \quad (5.5)$$

Here $\delta(i, j)$ is the Kronecker delta function, and the terms in the summation are Wigner $3j$ symbols. Armed with Eq. (5.5), we can calculate the matrix of \hat{H} in a truncated basis (working up to some cutoff values of J and L), and find the lowest few eigenvalues. Of course the system is rotation-

ally invariant, and hence the matrix block diagonalizes into blocks of good total angular momentum and good z component of the total angular momentum. In principle, one can exploit this symmetry to greatly reduce the size of the matrices considered. It is quite straightforward to exploit the

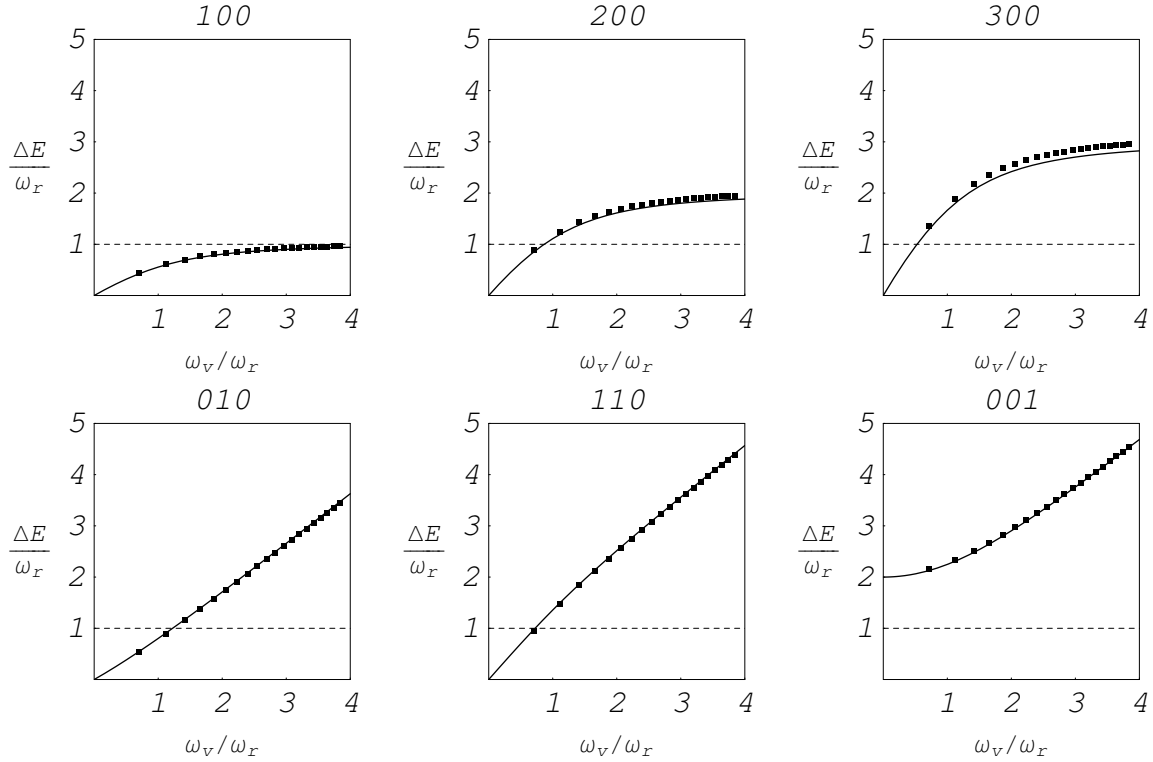


FIG. 2: We plot the numerical simulation data (dots) against the semiclassical approximation solutions based on the Callan-Klebanov approach (solid lines) for each of the energy levels shown in Fig. 1. The plots are of $\Delta E = E - E_{\text{ground}}$. The numerical computation was done with $qg = 40$ and $J_{\text{max}} = L_{\text{max}} = 48$.

third component of the total angular momentum: to find the spectrum it is sufficient to study states of total $m = 0$ since all multiplets have an $m = 0$ member. Since the z component of the angular momentum is additive it is sufficient to study basis states which have $m_J = -m_L$. Imposing good total angular momentum is in principle straightforward, but in practice is rather cumbersome due to the large number of terms in the Clebsch-Gordan series (as a result of the large cutoffs needed for convergence for the cases of numerical interest). Thus it was simpler to work with the full matrices for total $m = 0$. Taking $qg = 40$ to ensure large N_c , our matrices were of dimension approximately 21,000. Fortunately they are quite sparse and easily amenable to sparse matrix techniques. With these methods the lowest several eigenvalues of the system were easily calculated.

In Fig. 2, numerical solutions for several low-lying energy levels are presented for the case $qg = 40$. They are com-

pared to the semiclassical predictions based on the Callan-Klebanov approach; a wide variety of spring constants are considered. The plots are expressed in terms of the same dimensionless ratios as used in Fig. 1. To keep the graphs uncluttered, we have presented each energy level as a function of the strength of the spring constant on a separate graph. We used the level ordering given in the semiclassical expression, Eq. (4.3), to associate particular numerically computed energies with semiclassical states; *i.e.*, the n^{th} lowest numerical computed state is associated with the n^{th} lowest state in Eq. (4.3). There is some ambiguity with this method in the immediate vicinity of level crossings in which case we used numerical smoothness to determine the association of levels. It is quite apparent that the actual energy levels of the system closely follow the semiclassical treatment based on the Callan-Klebanov approach. Moreover, generically they are rather far from the prediction of rigid rotor quantiza-

tion; namely, that a collective state should exist at $\frac{\Delta E}{\omega_r} = 1$ (which is indicated in both Fig. 1 and Fig. 2 as a dashed line).

Of course, the semiclassical predictions based on the Callan-Klebanov method are not expected to precisely reproduce the spectra. One expects $1/N_c$ corrections with a characteristic scale of $(qg)^{-1}$ and that such corrections will increase with increasing excitation energy due to anharmonicities. Thus it seems highly plausible that the small but discernible deviations of the semiclassically predicted 200 and 300 modes from the numerical simulation are due to such effects. Indeed it is easy to see that the scale of these deviations is typical of what one expects. While it is generically nontrivial to compute the $1/N_c$ corrections, it is quite straightforward to do so in the limit $\frac{\omega_v}{\omega_r} \rightarrow \infty$, which is the infinitely strong coupling limit. In this case, the dynamics undoubtedly do reduce to that of a single particle of mass $2M$, and Eq. (3.2) holds for any N_c . This implies that in the limit $\frac{\omega_v}{\omega_r} \rightarrow \infty$ the actual value for $\frac{\Delta E}{\omega_r}$ in a state $n00$ will exceed the Callan-Klebanov prediction by an amount given by $\frac{n^2+1}{2qg}$. This is clearly a $1/N_c$ correction. It gives a correction of 0.025, 0.0625 and 0.125 for the 100, 200 and 300 states, respectively, in the strong coupling limit. For the largest values of $\frac{\omega_v}{\omega_r}$ we computed ($\frac{\omega_v}{\omega_r} = 3.84$), the actual amount by which the Callan-Klebanov formula underpredicted the numerical result was 0.0250, 0.0739, and 0.147, respectively, and it is highly plausible that these values will asymptote to the known $1/N_c$ corrections in the strong coupling limit. Thus, the size of the deviations from the Callan-Klebanov predictions is of the scale expected from $1/N_c$ corrections.

Upon the completion of this work we learned of a calculation of the spectrum of this model by Diakonov and Petrov. Their results agree with ours—the Callan-Klebanov results accurately describe the spectrum while the rigid rotor does not[16].

VI. CONCLUSION

The analysis of the toy system considered above shows explicitly that for a system with topological velocity-dependent interactions analogous to a dynamically active Wess-Zumino term, the Callan-Klebanov method is the correct way to implement semiclassical quantization. On the other hand, rigid-rotor quantization does not generally work in this system.

It is worth understanding why the plausible sounding argument given in Sect. III for rigid-rotor quantization fails.

The key point is that the intuition gained from the case without the monopole—that the strong coupling present at large N_c implies that the center of mass of the system moves collectively—does not automatically translate to the case where the monopole *is* present. While the strength of the coupling remains large, the effect of the monopole on the internal dynamics is also parametrically large for large N_c . The reason that the monopole plays such a role should be clear. If one imagines the center of mass of the composite system slowly moving in an apparently collective way, one finds that the monopole exerts a force only on the charged constituent, but *not* on the neutral one. The neutral constituent can only follow the charged one due to the force exerted by the spring. This implies possible mixing of the internal dynamics associated with the spring and the collective dynamics. Such mixing will be strong if the characteristic scale of the internal vibrations ω_v is comparable to the scale associated with collective motion ω_r . As both of these scales are parametrically of order N_c^0 there is no reason associated with $1/N_c$ physics for them not to mix strongly and thereby ruin the rigid rotor dynamics. This is the semiclassical argument outlined in ref. [8]. Underlying this is the simple time scale argument of ref. [2].

An alternative way to see this is simply to look at the classical dynamics of the system discussed in the appendix. The essential point there is to note that although there are two center of mass degrees of freedom (say, moving in the x and y directions from the north pole) there is only *one* zero mode and it is purely static. Rigid-rotor quantization is only legitimate for true collective motion associated with zero modes. Since there is no dynamical zero mode in the problem, one does not expect rigid-rotor quantization to apply. It is worth noting here that the analogous thing occurs for the chiral soliton case: it is precisely the lack of dynamical zero modes which accounts for the difference between the Callan-Klebanov and rigid-rotor approaches.

Of course, the model considered here is just a toy. The real question of interest is what form of quantization is correct for the soliton models. However, the analogy with the actual models is, in fact, very close: the key issues of topology, velocity-dependent forces and the existence of collective and vibrational modes with the same quantum numbers are present in both the toy problem and in the problem of physical interest. Thus, this calculation should be viewed as strong evidence that in the physically interesting problems arising in soliton models the Callan-Klebanov approach is correct at large N_c and the rigid-rotor approach is generally incorrect.

There are models for which the rigid rotor approximation does work, such as the model considered in ref. [11]. However, in that case it is easy to see that the Callan-Klebanov approach gives the same result as the rigid rotor method at large N_c . [10] Thus, the present situation is one in which the Callan-Klebanov approach has been shown to be valid at large N_c for all cases studied, while the rigid rotor method is only valid when it agrees with the Callan-Klebanov approach. This strongly suggests that where the two methods disagree the Callan-Klebanov quantization is the correct one. Since the two approaches are known to disagree for chiral soliton models[9], it seems highly unlikely that the rigid rotor quantization is valid; at large N_c Callan-Klebanov quantization is the correct approach. This strongly suggests that the many calculations done using rigid rotor quantization for exotic states of $SU(3)$ solitons are unjustified.

Acknowledgments

The authors acknowledge constructive comments from I. Klebanov, D. Diakonov, D. Dakin and P. V. Poblitsa.

This work was supported by the U.S. Department of Energy through grant DE-FG02-93ER-40762. One of the authors (AN) acknowledges the support of the University of Maryland through the Senior Summer Scholars program.

VII. APPENDIX

Here we review the semiclassical treatment of the toy problem, following ref. [8]. The first step is the treatment of classical motion at small amplitude centered on the north pole. For small amplitude motion, the system looks like a particle on a plane with a constant perpendicular magnetic field. In this regime, we can write the linearized equations of motion for the particles and their velocities to first order as

$$\frac{d\mathbf{v}}{dt} = i\overline{M}\mathbf{v} \quad (7.1)$$

with

$$\overline{M} = -i \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{\omega_v^2}{2} & 0 & \frac{\omega_v^2}{2} & 0 & 0 & 2\omega_r & 0 & 0 \\ 0 & -\frac{\omega_v^2}{2} & 0 & \frac{\omega_v^2}{2} & -2\omega_r & 0 & 0 & 0 \\ \frac{\omega_v^2}{2} & 0 & -\frac{\omega_v^2}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\omega_v^2}{2} & 0 & -\frac{\omega_v^2}{2} & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} x_q \\ y_q \\ x_n \\ y_n \\ \dot{x}_q \\ \dot{y}_q \\ \dot{x}_n \\ \dot{y}_n \end{pmatrix} \text{ with } \omega_v = \sqrt{\frac{2k}{m}}, \quad \omega_r = \frac{gq}{2mR^2}. \quad (7.2)$$

The subscripts q and n refer to the charged and neutral particles respectively, and x and y refer to the Cartesian coordinates. Note that since there are velocity-dependent terms it is natural to work in terms of coupled first-order differential equations. The canceling factors of i in Eqs. (7.1) and (7.2) are put in for later convenience.

We are interested in finding the eigenmodes of the system; *i.e.*, harmonic solutions of the form $\mathbf{v}(t) = \mathbf{v}_j \exp(-i\omega_j t)$. Inserting this ansatz into Eq. (7.1) yields a simple eigenvalue

equation:

$$\omega_j \mathbf{v}_j = \overline{M} \mathbf{v}_j. \quad (7.3)$$

On physical grounds, we know the ω_j are real. Moreover the matrix \overline{M} is purely imaginary which implies that if \mathbf{v}_j is an eigenvector with eigenvalue ω_j then \mathbf{v}_j^* is an eigenvector with eigenvalue $-\omega_j$. Thus, the eigenvectors form pairs associated with positive and negative frequency solutions. We refer to one of these pairs together as an eigenmode.

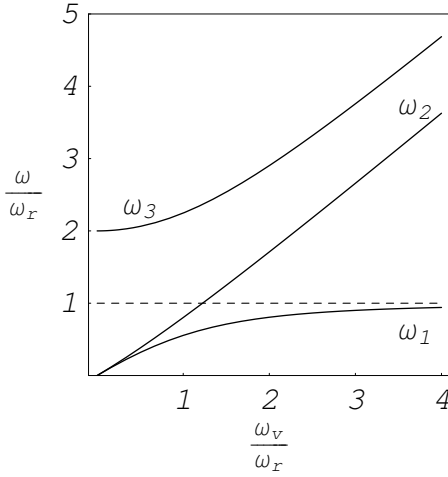


FIG. 3: We plot the non-zero semiclassical eigenmodes of the system, ω_1 , ω_2 , ω_3 .

One of these modes is a zero frequency mode:

$$\mathbf{v}_0 = \begin{pmatrix} 1 \\ i \\ 1 \\ i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (7.4)$$

A striking feature of this zero mode is the absence of any time dependence in it: the four lower components associated with the time derivatives are zero. Thus, this mode is associated with pure static rotations. By extracting the real and imaginary parts (\mathbf{v}_0 and \mathbf{v}_0^* are degenerate since $\omega = 0$ and hence one can form linear combinations of the two) we see that this mode corresponds to a collective time-independent rotation of the two particles in either the x or y directions. As this mode is completely non-dynamical, when quantized it is associated with “excitations” which move from one of the highly degenerate ground states of the theory to another. In this context it is useful to recall that the degeneracy of the ground state is $2qg + 1$ which diverges at large N_c .

It is extremely important to note that the one zero mode found above is the *only* zero mode for the system. We can solve Eq. (7.3) to find the three non-zero frequency eigenmodes of the system. The three eigenvalues can be computed analytically, but the form of the solutions is cumbersome and not particularly illuminating. We denote the three eigenfrequencies as ω_1 , ω_2 , and ω_3 defined such that $\omega_3 > \omega_2 > \omega_1$. These are plotted in terms of convenient combinations of variables in Fig. 3.

As these modes are harmonic in the large N_c limit, they can be quantized trivially yielding Eq. (4.3).

-
- [1] The first report was in T. Nakano *et al.*, Phys. Rev. Lett. **91**, 012002 (2003). A number of other groups have also reported evidence for the θ^+ . A summary of the situation as of the end of 2003 can be found in S. Eidelman *et al.*, Phys. Lett. **B 592**, 1 (2004).
 - [2] T. D. Cohen, Phys.Lett. B **581**, 175-181 (2004).
 - [3] M. Praszalowicz, talk at “Workshop on Skyrmions and Anomalies”, M. Jezabek and M. Praszalowicz, editors (World Scientific, 1987), page 112. This work was not published in a journal at that time. An updated analysis based on this work may be found in hep-ph/0308114.
 - [4] D. Diakonov, V. Petrov and M. Polyakov, Z. Phys. A **359**, 305 (1997).
 - [5] E. Guadagnini, Nucl. Phys. B **236**, 35 (1984).
 - [6] P. O. Mazur, M. A. Nowak and M. Praszalowicz, Phys. Lett. B **147**, 137 (1984); A. V. Manohar, Nucl. Phys. B **248**, 19 (1984); M. Chemtob, Nucl. Phys. B **256**, 600 (1985); S. Jain and S. R. Wadia, Nucl. Phys. B **258**, 713 (1985).
 - [7] G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B **228**, 522 (1983).
 - [8] T. D. Cohen, hep-ph/0312191.
 - [9] N. Itzhaki, I. R. Klebanov, P. Ouyang, L. Rastelli, hep-ph/0309305.
 - [10] P. V. Pobylitsa, hep-ph/0310221.
 - [11] D. Diakonov and V. Petrov, Phys.Rev. D **69**, 056002 (2004).
 - [12] C. G. Callan, Jr. and I. R. Klebanov, Nucl. Phys. B **262**, 365 (1985); C. G. Callan, Jr., K. Hornbostel and I. R. Klebanov, Phys. Lett. B **202**, 269 (1988).
 - [13] B.-Y. Park, M. Rho and D.-P. Min hep-ph/0405246.
 - [14] E. Witten, Nucl. Phys. B **223**, 422 (1983).
 - [15] This is a standard topic in quantum mechanics; see, for example, L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, (Pergamon Press, Oxford, 1977).
 - [16] Dimitri Diakonov, private communication.